

Sketch the graph of an example of a function that satisfies all the following conditions.

SCORE: _____ / 2 PTS

The domain of the function is $[-5, -1) \cup (-1, 5]$

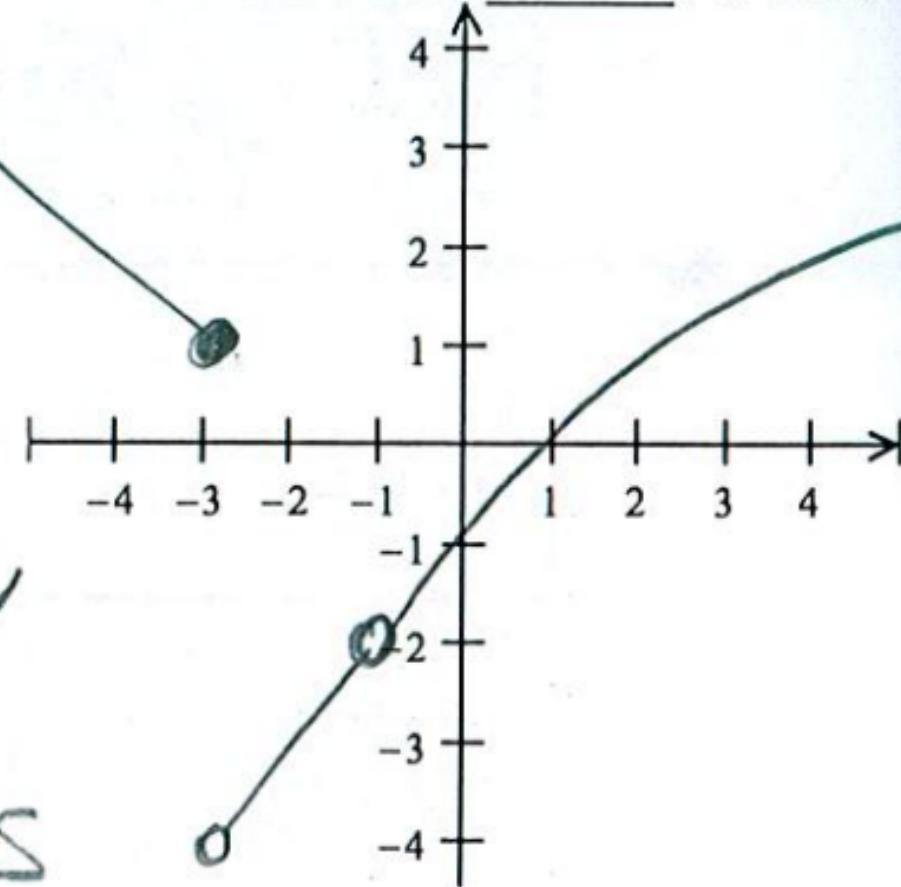
$$\lim_{x \rightarrow -3^-} f(x) = 1$$

$$\lim_{x \rightarrow -3^+} f(x) = -4$$

$$\lim_{x \rightarrow -1} f(x) = -2$$

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BY ME

INFINITELY
MANY
SOLUTIONS



$$\lim_{x \rightarrow 0} \sqrt[3]{x} \sin \frac{1}{x^2} = 0$$

Prove that

SCORE: _____ / 5 PTS

①
$$-1 \leq \sin \frac{1}{x^2} \leq 1$$
 FOR ALL $x \neq 0$

①
$$-\sqrt[3]{x} \leq \sqrt[3]{x} \sin \frac{1}{x^2} \leq \sqrt[3]{x}$$

①
$$\lim_{x \rightarrow 0} -\sqrt[3]{x} = \lim_{x \rightarrow 0} \sqrt[3]{x} = 0$$

① By SQUEEZE THEOREM, $\lim_{x \rightarrow 0} \sqrt[3]{x} \sin \frac{1}{x^2} = 0$ ①

The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist.

SCORE: _____ / 4 PTS

[a] $\lim_{x \rightarrow 3} \frac{x}{[f(x)]^2 - 2}$ ← Show the proper use of limit laws to find your answer.

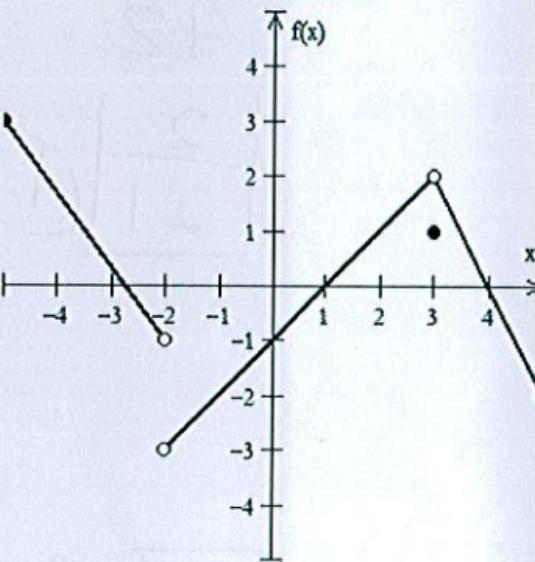
$$= \left[\frac{\lim_{x \rightarrow 3} x}{\lim_{x \rightarrow 3} f(x) \lim_{x \rightarrow 3} f(x) - \lim_{x \rightarrow 3} 2} \right]$$

$$= \left[\frac{3}{2^2 - 2} \right] \textcircled{1/2} \quad \textcircled{2}$$

$$= \left[\frac{3}{2} \right] \textcircled{1/2}$$

[b] $\lim_{x \rightarrow -2^-} f(x)$

$$= \boxed{-1}$$



$$\lim_{x \rightarrow -1} f(x) \text{ if } f(x) = \begin{cases} \sqrt{x+5}, & \text{if } x > -1 \\ 2, & \text{if } x = -1 \\ \frac{4}{x-1}, & \text{if } x < -1 \end{cases}$$

$$\lim_{x \rightarrow -1^-} f(x) = \boxed{\lim_{x \rightarrow -1^-} \frac{4}{x-1} = \frac{4}{-2} = -2}$$

$$\lim_{x \rightarrow -1^+} f(x) = \boxed{\lim_{x \rightarrow -1^+} \sqrt{x+5} = \sqrt{4} = 2}$$

$$\lim_{x \rightarrow -1} f(x) \text{ DNE}$$

$$\lim_{x \rightarrow 2} \frac{\sqrt{5x-1} - 3}{6 - 3x} \quad \text{0/0 INDETERMINATE}$$

$$= \lim_{x \rightarrow 2} \frac{(\sqrt{5x-1} - 3)(\sqrt{5x-1} + 3)}{(6 - 3x)(\sqrt{5x-1} + 3)}$$

$$= \boxed{\lim_{x \rightarrow 2} \frac{5x-1 - 9}{(6-3x)(\sqrt{5x-1} + 3)}} \quad (1)$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{5x+10}^5}{(\cancel{6-3x})(\sqrt{5x-1} + 3)}$$

$$= \boxed{\lim_{x \rightarrow 2} \frac{5}{-3(\sqrt{5x-1} + 3)}} \quad (1)$$

$$= \frac{5}{-3 \cdot 6} = \boxed{-\frac{5}{18}} \quad (1)$$

$$\lim_{x \rightarrow -4} \frac{\frac{6}{2-x} - 1}{2 + \frac{8}{x}} \quad \frac{0}{0} \text{ INDETERMINATE}$$

$$= \lim_{x \rightarrow -4} \left(\frac{6-2+x}{2-x} \div \frac{2x+8}{x} \right)$$

$$= \boxed{\lim_{x \rightarrow -4} \frac{4+x}{2-x} \cdot \frac{x}{2x+8}} \quad \textcircled{1}$$

$$= \boxed{\lim_{x \rightarrow -4} \frac{x}{2(2-x)}} \quad \textcircled{1}$$

$$= \frac{-4}{12}$$

$$= \boxed{-\frac{1}{3}} \quad \textcircled{1}$$

$$\lim_{x \rightarrow -3} \frac{3x^2 + 7x - 2}{2x^2 - 7x + 3}$$

$$= \frac{3(-3)^2 + 7(-3) - 2}{2(-3)^2 - 7(-3) + 3}$$

$$= \frac{27 - 21 - 2}{18 + 21 + 3}$$

$$= \frac{4}{42}$$

$$= \boxed{\frac{2}{21}} \textcircled{1}$$